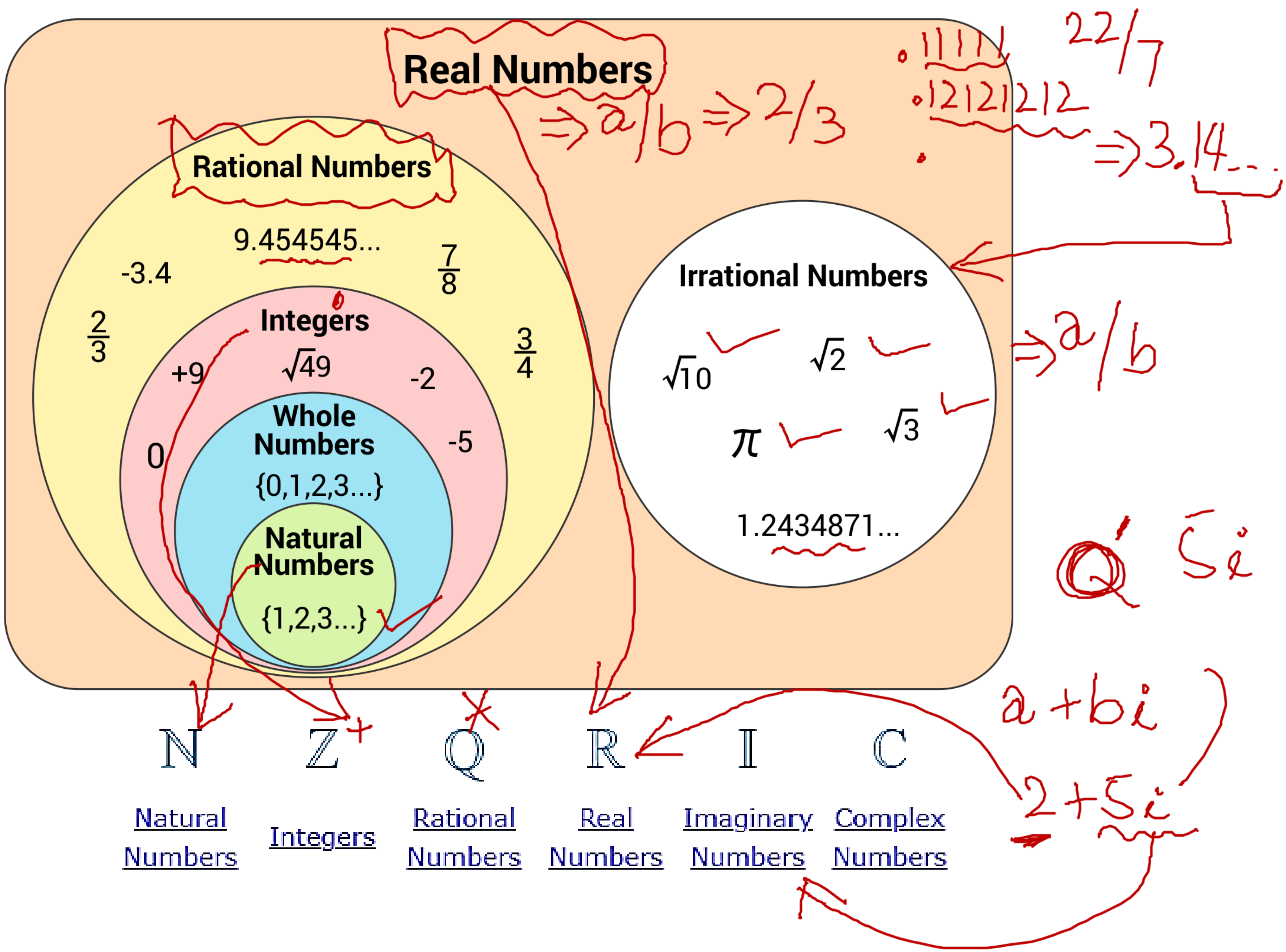
Set Theory

A **set** is an unordered collection of distinct objects, called elements or members of the set. A set is said to contain its elements. We write $a \in A$ to denote that a is an element of the set A. The notation $a \notin A$ denotes that a is not an element of the set A.

- The set V of all vowels in the English alphabet can be written as V = {a, e, i, o, u}
- The set O of odd positive integers less than 10 can be expressed by O = {1, 3, 5, 7, 9}.
- The set of positive integers less than 100 can be denoted by {1, 2, 3, … , 99}


Real Numbers
Rational Numbers
9.454545...
-3.4
7/8
2/3
3/4
Integers
+9
√49
-2
-5
0
Whole Numbers
{0,1,2,3...}
Natural Numbers
{1,2,3...}
Irrational Numbers
√10
√2
π
√3
1.2434871...
N
Z
Q
R
I
C
Natural Numbers
Integers
Rational Numbers
Real Numbers
Imaginary Numbers
Complex Numbers

How to describe a set by saying what properties its members have.

Descriptive Form	Set - Builder Form	Roster Form
The set of all vowels in English alphabet	$\{x \mid x$ is a vowel in the English alphabet$\}$	$\{a, e, i, o, u\}$
The set of all odd positive integers less than or equal to 15	$\{x \mid x$ is an odd number and $0 < x \leq 15\}$	$\{1, 3, 5, 7, 9, 11, 13, 15\}$
The set of all positive cube numbers less than 100	$\{x \mid x$ is a cube number and $0 < x < 100\}$	$\{1, 8, 27, 64\}$

$$O = \{x \mid x \text{ is an odd positive integer less than } 10\},$$

or, specifying the universe as the set of positive integers, as

$$O = \{x \in \mathbf{Z}^+ \mid x \text{ is odd and } x < 10\}.$$

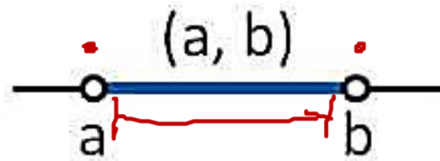
We often use this type of notation to describe sets when it is impossible to list all the elements of the set. For instance, the set $\mathbf{Q}^+$ of all positive rational numbers can be written as

$$\mathbf{Q}^+ = \{x \in \mathbf{R} \mid x = \tfrac{p}{q}, \text{ for some positive integers } p \text{ and } q\}.$$

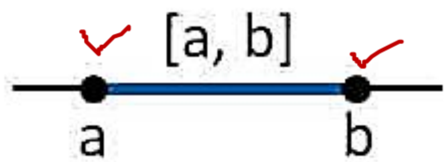
$O = \{1, 3, 5, 7, 9\}$

$\Rightarrow a/b$

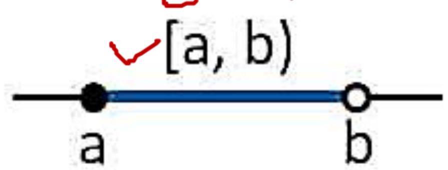
Intervals Notation



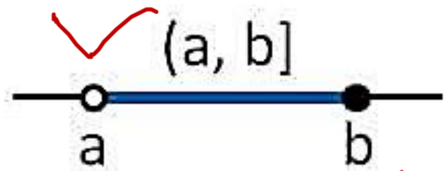
$a < x < b$, Open interval



$a \le x \le b$, Closed interval



$a \le x < b$, Semi Open interval



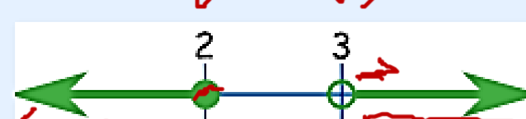
$a < x \le b$, Semi Open interval

Example: $x \le 2$ or $x > 3$

Set-Builder Notation looks like this:

$\{ x \in \mathbb{R} \mid x \le 2 \text{ or } x > 3 \}$

On the Number Line it looks like:



Using Interval notation it looks like:

$(-\infty, 2] \cup (3, +\infty)$

Types of Set

Finite Set

A set which contains a definite number of elements is called a finite set.

Example – S={ x | x∈N and 70>x>50}

Infinite Set

A set which contains infinite number of elements is called an infinite set.

Example – S={ x | x∈N and x>10}

Subset

A set X is a subset of set Y (Written as X⊆Y) if every element of X is an element of set Y.

Example 1 – Let, X={1,2,3,4,5,6} and Y={1,2}.

Here set Y is a subset of set X as all the elements of set Y is in set X. Hence, we can write Y⊆X.

$$\forall x(x \in A \rightarrow x \in B)$$

Example 2 – Let, X={1,2,3} and Y={1,2,3}.

Here set Y is a subset (Not a proper subset) of set X as all the elements of set Y is in set X. Hence, we can write Y⊆X.

Types of Set

Proper Subset

The term “proper subset” can be defined as “subset of but not equal to”. A Set X is a proper subset of set Y (Written as X⊂Y) if every element of X is an element of set Y and |X|<|Y|.

Example – Let, X={1,2,3,4,5,6} and Y={1,2}. Here set Y⊂X since all elements in Y are contained in X too and X has at least one element is more than set Y.

$$\forall x(x \in A \rightarrow x \in B) \wedge \exists x(x \in B \wedge x \notin A)$$

Universal Set

It is a collection of all elements in a particular context or application. All the sets in that context or application are essentially subsets of this universal set. Universal sets are represented as U.

Example – We may define U as the set of all animals on earth. In this case, set of all mammals is a subset of U, set of all fishes is a subset of U, set of all insects is a subset of U, and so on.

Empty Set or Null Set

An empty set contains no elements. It is denoted by ∅. As the number of elements in an empty set is finite, empty set is a finite set. The cardinality of empty set or null set is zero.

Example – S={x|x∈N and 7<x<8}=∅

Types of Set

Singleton Set or Unit Set

Singleton set or unit set contains only one element.

A singleton set is denoted by {s}.

Example – S={x|x∈N, 7<x<9} = {8}

Equivalent Set

If the cardinalities of two sets are same, they are called equivalent sets.

Example – If A={1,2,6} and B={16,17,22},

they are equivalent as cardinality of A is equal to the cardinality of B. i.e. |A|=|B|=3

Equal Set

If two sets contain the same elements they are said to be equal.

Example – If A={1,2,6} and B={6,1,2}, they are equal as every element of set A is an element of set B and every element of set B is an element of set A.

$$\forall x (x \in A \leftrightarrow x \in B)$$

Another look at Equality of Sets

- Recall that two sets A and B are *equal*, denoted by $A = B$, iff

$$\forall x (x \in A \leftrightarrow x \in B)$$

- Using logical equivalences we have that $A = B$ iff

$$\forall x [(x \in A \rightarrow x \in B) \wedge (x \in B \rightarrow x \in A)]$$

- This is equivalent to

$$A \subseteq B \quad \text{and} \quad B \subseteq A$$

Types of Set

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Overlapping Set

Two sets that have at least one common element are called overlapping sets.

Example − Let, A={1,2,6} and B={6,12,42}.

There is a common element ‘6’, hence these sets are overlapping sets.

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Disjoint Set

Two sets A and B are called disjoint sets if they do not have even one element in common. Therefore, disjoint sets have the following properties −

$n(A \cap B) = \emptyset$

$n(A \cup B) = n(A) + n(B)$

Example − Let, A={1,2,6} and B={7,9,14}, there is not a single common element, hence these sets are disjoint sets.

What is the power set of the set {0, 1, 2}?

Solution: The power set ({0, 1, 2}) is the set of all subsets of {0, 1, 2}.
Hence, Examples ({0, 1, 2}) = {∅, {0}, {1}, {2}, {0, 1}, {0, 2}, {1, 2}, {0, 1, 2}}.
Note that the empty set and the set itself are members of this set of subsets

Cartesian product of A = {1, 2} and B = {a, b, c}?

Solution: The Cartesian product A × B is;
A × B = {(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)}.

$$A \times B = \{(a, b) | a \in A \wedge b \in B\}$$

The **cardinality** of a finite set A, denoted by |A|, is the number of (distinct) elements of A.

Examples: 1. |ø| = 0

2. Let S be the letters of the English alphabet. Then |S| = 26

3. |{1,2,3}| = 3

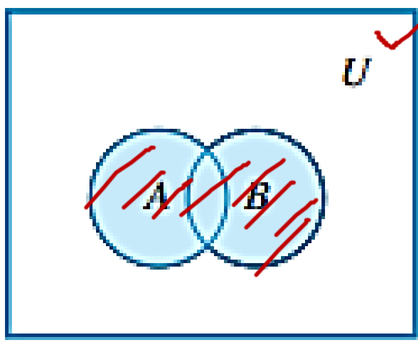
4. |{ø}| = 1

5. The set of integers is infinite.

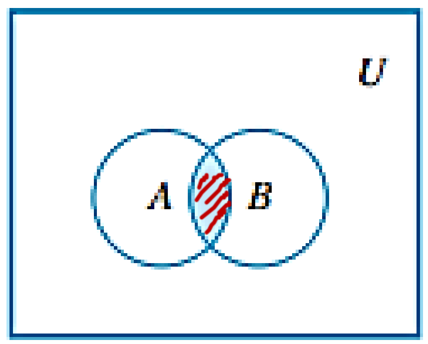
Set Operation

Venn Diagrams

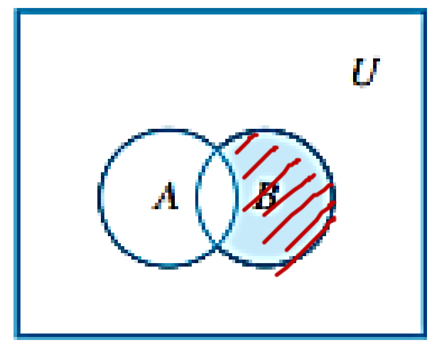
If sets A and B are represented as regions in the plane, relationships between A and B can be represented by pictures, called Venn diagrams, that were introduced by the British mathematician John Venn in 1881.



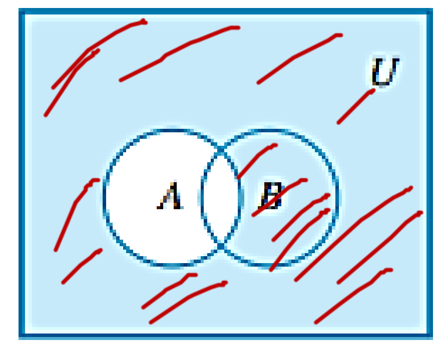
Shaded region represents $A \cup B$.



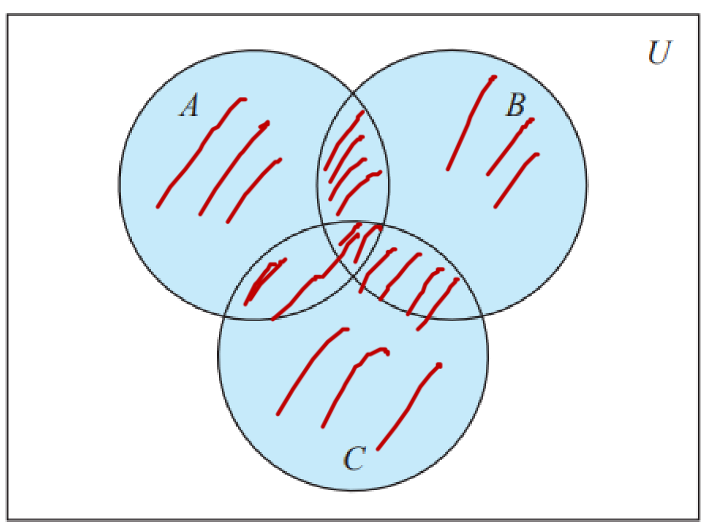
Shaded region represents $A \cap B$.



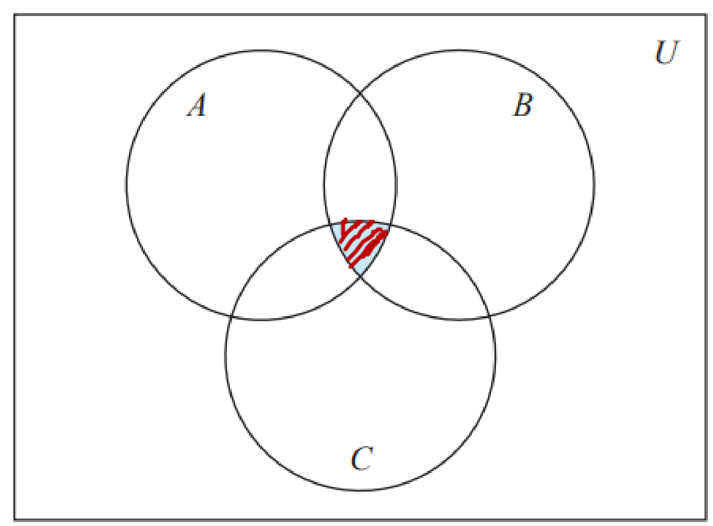
Shaded region represents $B - A$.



Shaded region represents A^c.



(a) $A \cup B \cup C$ is shaded.



(b) $A \cap B \cap C$ is shaded.

Set Operation

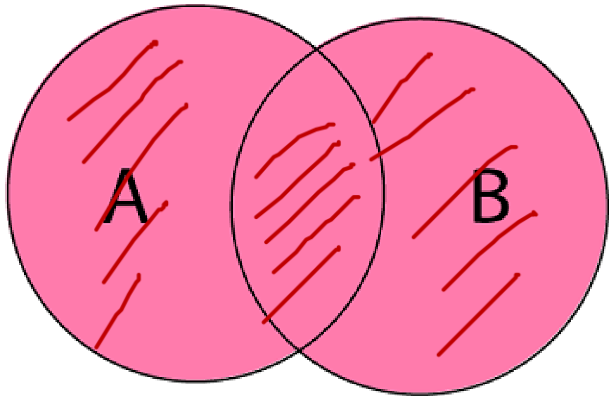
Union of Sets: Union of Sets A and B is defined to be the set of all those elements which belong to A or B or both and is denoted by AUB.

AUB = {x: x ∈ A or x ∈ B}

Example: Let A = {1, 2, 3}, B= {3, 4, 5, 6}

AUB = {1, 2, 3, 4, 5, 6}.

$$A \cup B = \{x \mid x \in A \vee x \in B\}.$$



|A ∪ B|=|A|+|B|−|A ∩ B|

Principle of inclusion–exclusion.

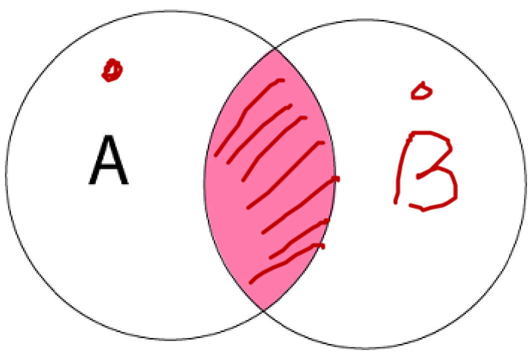
Intersection of Sets: Intersection of two sets A and B is the set of all those elements which belong to both A and B and is denoted by A ∩ B.

A ∩ B = {x: x ∈ A and x ∈ B}

Example: Let A = {11, 12, 13}, B = {13, 14, 15}

A ∩ B = {13}

$$A \cap B = \{x \mid x \in A \wedge x \in B\}.$$



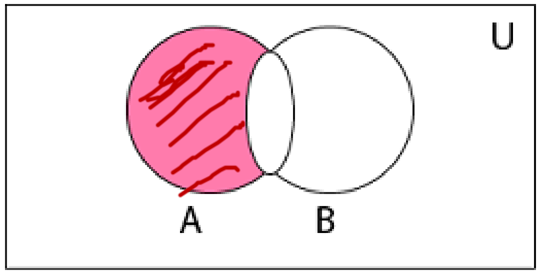
Difference of Sets: The difference of two sets A and B is a set of all those elements which belongs to A but do not belong to B and is denoted by A - B.

A - B = {x: x ∈ A and x ∉ B}

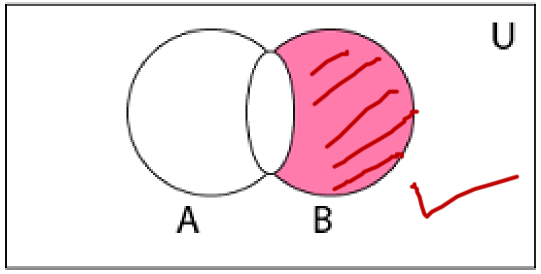
Example: Let A = {1, 2, 3, 4} and B = {3, 4, 5, 6}

then A - B = {3, 4} and B - A = {5, 6}

$$A - B = \{x \mid x \in A \wedge x \notin B\}.$$



A - B

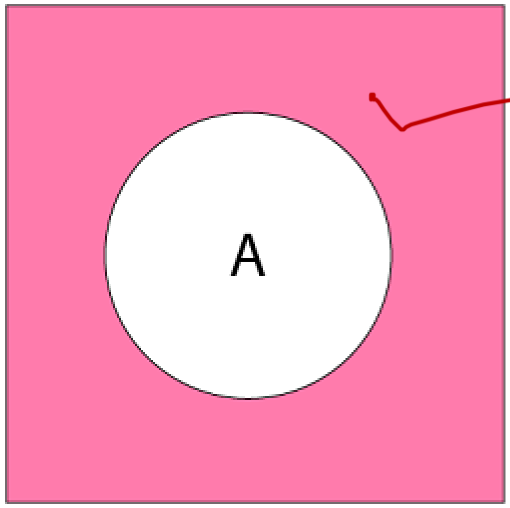


B - A

$$A - B = A \cap \overline{B}.$$

Set Operation

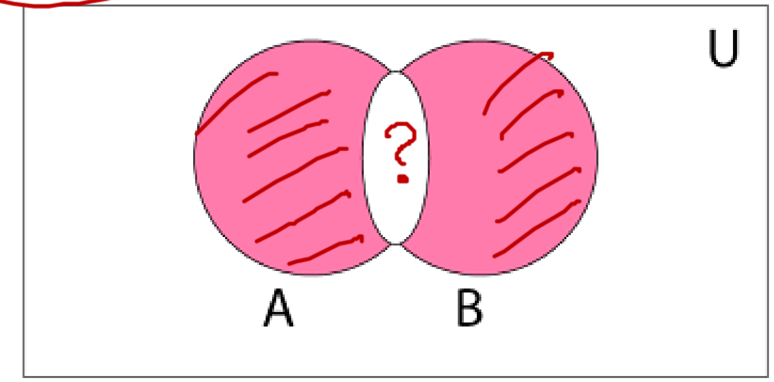
Complement of a Set: The Complement of a Set A is a set of all those elements of the universal set which do not belong to A and is denoted by Ac.



Ac = U - A = {x: x ∈ U and x ∉ A} = {x: x ∉ A}
Example: Let U is the set of all natural numbers.
A = {1, 2, 3}
Ac = {all natural numbers except 1, 2, and 3}.

$$\overline{A} = \{x \in U \mid x \notin A\}.$$

Symmetric Difference of Sets: The symmetric difference of two sets A and B is the set containing all the elements that are in A or B but not in both and is denoted by A ⊕ B i.e.



A ⊕ B = (A ∪ B) - (A ∩ B)
Example: Let A = {a, b, c, d}
B = {a, b, l, m}
A ⊕ B = {c, d, l, m}

$$A \triangle B = (A - B) \cup (B - A).$$