Assignment

1. **(Implement & Analyze)** Implement **Merge Sort** and **Quick Sort** in your preferred language. Use them to sort a randomly generated array of 1,000 integers.
   - Measure and record execution times for both algorithms at input sizes 100, 500, 1000, and 5000.
   - Plot a graph of running time vs. input size for both algorithms.
   - Analyze your results in comparison with their asymptotic time complexities.
2. **(Evaluate)** Formally prove the **loop invariants** for the merge step of Merge Sort. Clearly explain how these invariants guarantee correctness.
3. You are given the following functions describing running times of different algorithms:

- $f_1(n) = n^2 + 5n$
- $f_2(n) = 20n\log n + 7n$
- $f_3(n) = n!$

Tasks:

a) Rank these functions in order of increasing growth rate as $n \to \infty$, and explain your reasoning.

b) Determine the Big-O notation for each function and justify your answer.

c) Provide a practical example of a problem for which each growth rate could occur.